VAR: history or simulation?

Greg Lambadiaris, Louiza Papadopoulou, George Skiadopoulos and Yiannis Zoulis assess the performance of historical and Monte Carlo simulation in calculating VAR, using data from the Greek stock and bond market. They find that while historical simulation results in over-commitment of capital for linear stock portfolios, the results for non-linear bond portfolios are less clear

The value-at-risk of a portfolio is the maximum loss expected to occur with a certain probability over a given time period. Even though VAR is subject to a number of limitations, the Basel Committee on Banking Supervision has chosen it as the standard method to measure the market risk of a portfolio of financial assets.[1] Basel I allows financial institutions to use their own internal models to calculate VAR provided the models are accurate. There is much literature comparing various VAR models. However, there is no unanimous agreement on what VAR model should be preferred; the answer seems to be a function of the specified portfolio and the data set used to estimate the parameters (see, for example, Beder, 1995, and Hendricks, 1996). Furthermore, VAR is expected to remain unchanged as an approved regulatory methodology in the market risk component of Basel II, the forthcoming new bank capital adequacy Accord (see also Dunbar, 2002, for a discussion on the applicability of VAR as a risk measure). Hence, the question of which is the best method to calculate VAR is still valid.

In this article, we assess the performance of historical and Monte Carlo (MC) simulation as alternative approaches to calculating VAR, by using data from the Greek stock and bond market. To the best of our knowledge, this is the first time that data from the Greek markets has been used to compare various VAR methods. This allows examining non-parametric and parametric VAR methods in a turbulent market environment[2]; this is particularly important for deciding on which is the most reliable method.

For both approaches, the one-day VAR is calculated for two separate portfolios: a linear one consisting of stocks and a non-linear one consisting of bonds. Employing different types of portfolios is important since the performance of the method may depend on the portfolio's nature. VAR is calculated separately for a 99% and 95% confidence level so as to examine whether the confidence level choice affects the accuracy of the methods.

The historical simulation VAR is calculated using one-day rolling window of 100 and 252 observations (HS-100 and HS-252); several papers have found that the performance of the method depends on the sample size. For example, Hendricks (1996) and Vlaar (2000) found that historical simulation tends to yield more accurate estimates as the sample size increases. In the stock portfolio case, the MC simulation VAR is obtained by using the moving average (MA-MC), the exponentially weighted moving average (EWMA-MC) and diagonal BEKK volatility estimators (BEKK-MC). In the bond portfolio case, we calculate VAR via MC simulation by choosing a multi-factor version of Dothan's (1978) process; the process can also be viewed as being analogous to one used in the Libor market models (for example, Brace, Gatarek & Musiela, 1997). The volatility structure of the assumed interest rate process is estimated by principal components analysis (PCA). Our use of PCA for VAR purposes is novel; it parallels the use of PCA in implementing Heath, Jarrow & Morton (1992) or Brace, Gatarek & Musiela (1997) type of models for option pricing purposes (see Singh, 1997, for an alternative use of PCA for calculating VAR). Finally, various standard criteria are used so as to decide on the validity of each VAR model (back-testing). The results are mixed and they have important implications for effective risk management.

The rest of the paper is organised as follows. In the next section, the historical and the MC simulation methods to calculate VAR are described. Various back-testing methods that have been proposed are also outlined. Then, we describe the data sets and the two methods are applied to the stock and bond portfolio, respectively. Back-testing is performed. The final section concludes and discusses the implications of the study.

Calculating and back-testing VAR

The α% T-period VAR is defined as the portfolio loss x in market value over the time horizon T that is not expected to be exceeded with probability $(1 - a)$, that is:

$$\Pr ob\left(\Delta_T \Pi_t \le x\right) = 1 - a \tag{1}$$

where $\Delta_T \Pi_t = \Pi_{t+T} - \Pi_t$ is the change in the portfolio value (profit/loss, P/L) over the holding period T. The portfolio value Π_t at time t is a function of the nominal amount W_i invested in the ith asset, and the price P_{it} of the ith asset at time t, that is:

$$\Pi_t = f\left(W_i, P_{it}\right), \qquad i = 1,\ldots,n \tag{2}$$

where n is the number of assets in the portfolio. Equivalently, the portfolio value in equation (2) may also be expressed in terms of the factors that affect the portfolio price (market factors).

By definition, the T-period VAR calculation requires constructing the T-period P/L portfolio's distribution. Two of the most commonly used methods to calculate VAR are the historical and the MC simulation (see Jorion, 2001, and Linsmeier & Pearson, 2000, for a thorough description). The historical simulation is a non-parametric method: it constructs the P/L portfolio's distribution by assuming that the changes in the portfolio value experienced in the past will prevail in the future. The MC simulation is a parametric method: it assumes that a certain process governs the dynamics of the market factors. Paths of the underlying market factors are simulated M times (M simulation runs) up to time T. The portfolio price Π_{Tj} ($j = 1, \ldots, M$) is evaluated, and the P/L portfolio distribution is constructed by subtracting Π_{Tj} from the current marked-to-market portfolio price.

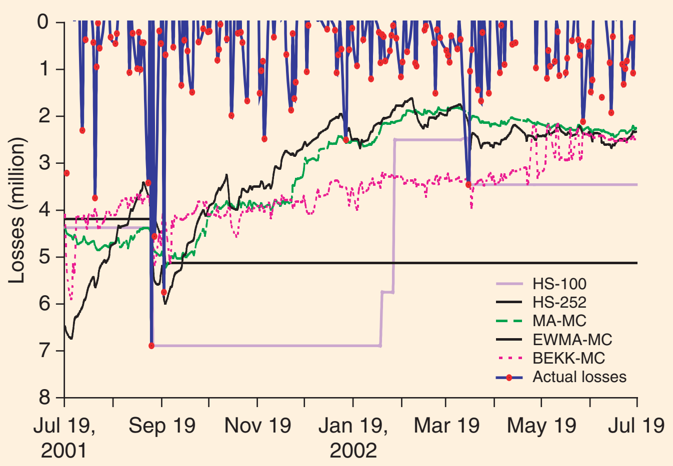
The ultimate test of every VAR model lies in its ability to forecast accurately the maximum loss likely to occur over the given probability level and time horizon. At the same time, the model should not tie up an excessive amount of capital. The process of assessing the forecasting performance is called back-testing. The simplest way to back-test the model is to check whether the number of cases that the actual losses exceed the calculated VAR (exceptions) equals the specified confidence level $(1 - \alpha)$. This is the Bank for International Settlements (BIS) guideline.[3] However, since the sam-

[1] VAR has been criticised mainly on the grounds that it is not a coherent measure of risk (Artzner et al, *1997) and that it cannot quantify the losses that might be suffered beyond the amount indicated by this measure. An alternative measure of risk that overcomes these problems is conditional value-at-risk (CVAR), also known as expected shortfall. CVAR is defined as the expected value of the losses that exceed VAR. However, its accurate estimation is problematic due to the lack of sufficient amount of data in the tails of the profit/loss distribution*

[2] The Greek stock exchange undergoes market stretch frequently; the Greek General Index has dropped about 4,000 points in the past three years. The Greek bond market was affected by the convergence of the euro area interest rates dictated by the EMU framework

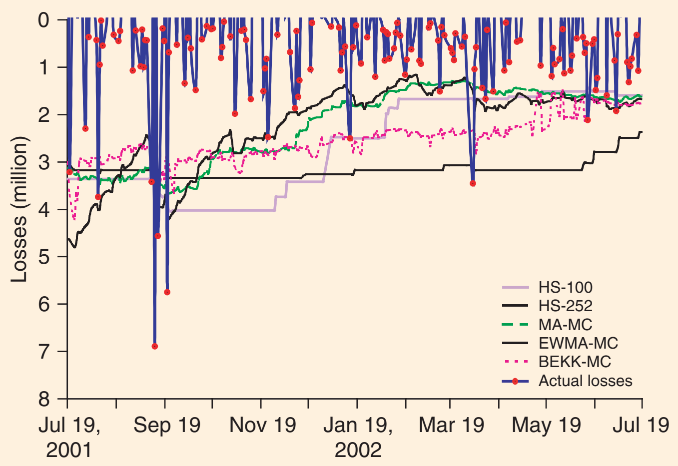
[3] According to the Basle Committee, each bank must meet a capital requirement expressed as the maximum of the bank's previous day's VAR number and an average of the daily VAR measures on each of the preceding 60 trading days, adjusted by a multiplication factor. The multiplication factor is to be set within a range of three to four depending on the supervisor's assessment of the bank's risk management practices and the results of a simple back-test that counts the number of exceptions. For a 250-day back-testing period and a 99% confidence level, a model that provides up to four exceptions falls in the 'green zone', receiving a multiplication factor of three. A model with five to nine exceptions falls in the 'yellow zone', with the multiplication factor increasing gradually up to 3.85. For 10 or more exceptions, the model falls in the 'red zone', where the multiplication factor is set equal to four

1. Stock portfolio (99%)



The 99% one-day HS-100 VAR, HS-252 VAR, MA-MC VAR, EWMA-MC VAR and Diagonal BEKK-MC VAR, and the actual losses over the back-testing period

2. Stock portfolio (95%)



The 95% one-day HS-100 VAR, HS-252 VAR, MA-MC VAR, EWMA-MC VAR and Diagonal BEKK-MC VAR, and the actual losses over the back-testing period

ple size of daily observations is finite, the actual number of exceptions may differ from the percentage implied by the model's confidence level, even in cases where the model is in fact accurate. Therefore, the accuracy of the model should be examined using various additional tests.

We employ the standard Kupiec (1995) 'proportion of failures' and Crnkovic & Drachman (1996) tests. The latter is also known as Kuiper test. The null hypothesis in the Kupiec test is that the number of exceptions equals $1 - \alpha$. Under the null hypothesis, the test follows a chi-squared distribution with one degree of freedom. The Crnkovic & Drachman test uses Kuiper's goodness-of-fit statistic to measure the distance between the entire probability distribution forecast of the portfolio's P/L and the actual P/L distribution. The smaller the value of Kuiper statistic, the more accurate the probability distribution forecast. The 95% confidence level cut-off values of the tests for the chosen parameters are 0.109 and 3.841, respectively (Jorion, 2001).

The data sets

The raw stock and bond dataset was obtained from Reuters for the period July 17, 2000 to July 18, 2002. For the stock portfolio case, the daily closing stock prices of five companies that are listed and traded in the Athens Stock Exchange are used. These companies are Alpha Bank, Coca-Cola Hellenic Bottling Company, Hellenic Telecom Organization, National Bank and Titan Cement. Based on those five stocks, an equally weighted portfolio is constructed.

For the bond portfolio analysis, bond prices as well as zero interest rates are required. We use daily Greek government bond prices with 10, 15 and 20 years maturity (at issue) paying coupons of 6%, 6.5% and 6.5%, respectively; the bonds are traded in the Greek government bond market. Euribor interest rates with maturities of one week, one, three, six and nine months, and one year, as well as swap rates with maturities of two, three, five, seven, 10, 12 and 20 years are used. The swap reset dates are semi-annual. A bond portfolio with a 50% weight of the 10-year bond and 25% weights of the 15- and 20-year bonds, respectively, is constructed.

The value of each portfolio is assumed to be €100 million. The weight of each portfolio asset, as well as the portfolio value, is assumed to remain constant throughout the out-of-sample period. Hence, rebalancing of the number of assets in the portfolio occurs daily.

To implement the VAR models, we split the initial sample into two subsamples. The first covers a one-year period (July 17, 2000 to July 18, 2001, 252 observations) and it is used to calculate the required inputs (for example, stock volatilities and correlations). The second contains the remaining 247 observations from July 19, 2001 to July 18, 2002 and will be used for the back-testing purposes (out-of-sample period). The choice of the length of the out-of-sample period is in accordance with the Basle Committee requirements.

The historical simulation is performed using a one-day rolling window of 100 and 252 observations and the MC simulations are performed using 15,000 simulation runs (Vlaar, 2000, has found that more than 10,000 simulation runs should be used to calculate the VAR of fixed-income portfolios). The one-day horizon VAR is calculated separately for a 99% and 95% confidence level.

The stock portfolio: results

In the stock portfolio case, the historical and MC simulation methods are applied to the stock portfolio prices. To calculate VAR by MC simulation, we assume that the stock price follows a geometric Brownian motion. The drift is set equal to zero; this is standard practice when the VAR horizon is short. The volatility and correlation coefficients are updated daily, estimated by three different models: a moving average (MA), an exponentially weighted moving average (EWMA) and a diagonal BEKK Garch(1, 1) model (see Engle & Kroner, 1995). The same stream of random numbers is used so the VAR results are comparable across the three estimators. Cholesky decomposition is applied so as to create correlated random numbers.

We follow Figlewski's (1997) methodology to decide on the number of observations to use in the MA estimator. The methodology consists of calculating the root percentage mean square error of volatility forecasts for different sample sizes and different forecasting horizons (10, 30, 60 and 74 days – the average return is set equal to zero). We find that a sample size of 74 observations yields the lowest root percentage mean square error for all forecasting horizons. In the EWMA model, the decay factor is set equal to 0.94 following RiskMetrics methodology (JP Morgan, 1996).

Figures 1 and 2 show the calculated 99% and 95% VAR, respectively, for the five methods (HS-100, HS-252, MA-MC, EWMA-MC and BEKK-MC) and the actual losses of the portfolio over the back-testing period. The various VAR patterns for the two confidence levels are quite similar. They differ in that in the 95% case, the VAR obtained from the HS-100 method evolves closer to the VAR obtained from the other methods, compared with the 99% case.

Table A summarises the results from the stock portfolio analysis for the various VAR methods. Both Kupiec and Kuiper tests accept the five VAR methods. In the 99% case, all methods yield almost the same number of exceptions and they fall in the 'green' BIS zone. In the 95% case, although

A. Stock portfolio

	HS-100	HS-252	MA-MC	EWMA-MC	BEKK-MC
99% VAR					
Average VAR	€4.932m	€4.964m	€3.165m	€3.189m	€3.554m
No. of exceptions	2	3	3	3	3
Kupiec test	0.097	0.108	0.108	0.108	0.108
Kuiper test	0.008	0.024	0.004	0.020	0.020
95% VAR					
Average VAR	€2.662m	€3.166m	€2.249m	€2.267m	€2.528m
No. of exceptions	10	7	12	8	8
Kupiec test	0.502	2.873	0.011	1.833	1.833
Kuiper test	0.008	0.024	0.004	0.020	0.020

Back-testing the one-day 99% and 95% HS-100 VAR, HS-252 VAR, MA-MC VAR, EWMA-MC VAR and Diagonal BEKK-MC VAR. Results are reported for Kupiec and Kuiper tests. The back-testing period is 247 days, and the total value of the portfolio is €100 million. The critical values for Kupiec and Kuiper tests are 3.841 and 0.109, respectively

the number of exceptions shows a greater variation across the various methods compared with the 99% case, the expected number of exceptions (that is, 12.5) is not exceeded either. However, the HS-252 method yields the most conservative average VAR figure for both the 95% and 99% case; this can be considered as a limitation of the specific method. Hence, MC simulation should be preferred to calculate the VAR of the stock portfolio.

The bond portfolio: results

In the bond portfolio case, the historical and the MC simulation are performed on the spot interest rates. Once the simulated zero curves are obtained, any risk-free bond can be priced by discounting its coupons; cubic spline interpolation is used to obtain the required for any horizon zero rates.

There are two main issues in implementing MC simulation on interest rates: the choice of the interest rate process and the method of estimating its parameters. An *n*-factor version of Dothan's (1978) interest rate process is chosen, that is:

$$dr_t(k) = r_t(k)\sum_{i=1}^{n} \nu_{k,i} dZ_{i,t} \qquad k = 1,2,\ldots,K \tag{3}$$

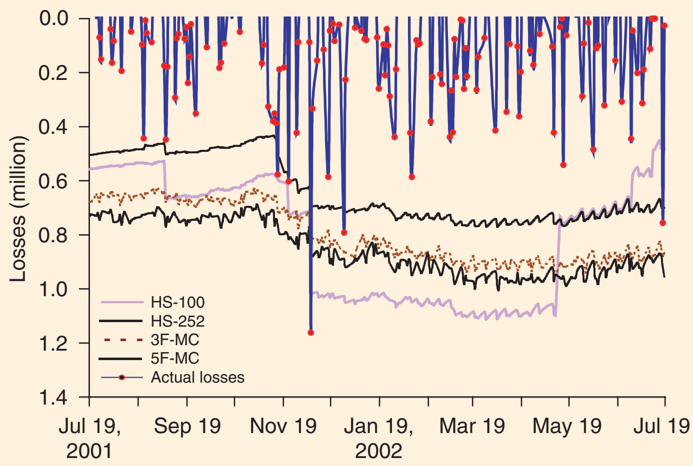
where $r_t(k)$ is the *k*-horizon spot interest rate and $\nu_{k,i}$ is its volatility, which corresponds to the Zi Brownian shock. Equation (3) rules out the presence of mean reversion in the interest rates. This is consistent with the results in Chan *et al* (1992), Brenner, Harjes & Kroner, (1996), and Nowman, 1997 (see also Chapman & Pearson, 2001, for an excellent survey). The use of equation (3) for VAR purposes is novel.[4]

We estimate the volatility structure and the number of shocks appearing in equation (3) by using PCA. This is the second contribution of this article to the VAR fixed-income literature. PCA is applied to the differences of zero interest rates across a spectrum of 10 maturities (that is, $K = 10$) and it provides a parsimonious way of calculating the VAR of a portfolio of fixed-income securities.[5] VAR is calculated over the out-of-sample period by updating the PCA estimates daily. Notice that there is no need to impose no-arbitrage constraints on the evolution of interest rates, and hence on the volatilities of the interest rates (for example, see Brace, Gatarek & Musiela, 1997, equation 2.5) since VAR is calculated under the physical probability measure.

We find that the first three principal components explain 80–88% of the total variation of the zero interest rates percentage changes throughout the back-testing period. To explain more than 95% of the total variance, five principal components need to be retained.[6] Hence, to calculate VAR we run two Monte Carlo simulations separately, using three (3F-MC method) and five principal components (5F-MC method), respectively.

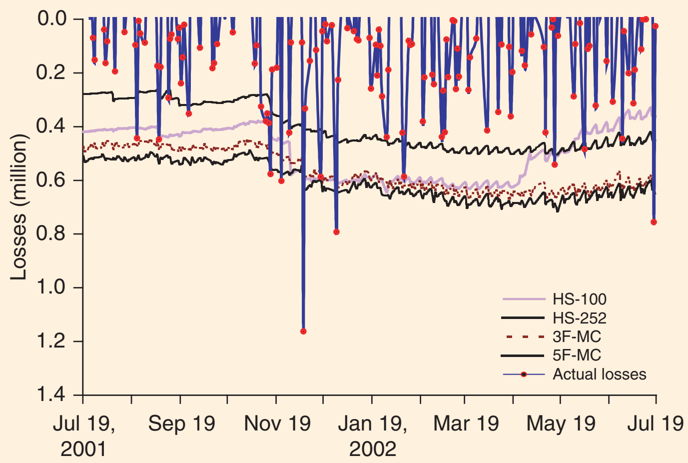
Figures 3 and 4 show the calculated 99% and 95% VAR, respectively, for the four methods (HS-100, HS-252, 3F-MC and 5F-MC) and the actual losses of the portfolio over the back-testing period.

3. Bonds portfolio (99%)



The 99% one-day HS-100 VAR, HS-252 VAR, 3F-MC VAR and 5F-MC VAR, and the actual losses over the back-testing period

4. Bonds portfolio (95%)



The 95% one-day HS-100 VAR, HS-252 VAR, 3F-MC VAR and 5F-MC VAR, and the actual losses over the back-testing period

In the 99% case, the estimated HS-252 VAR changes abruptly in November 2001. This is due to a sudden increase in the zero rates and hence a decrease of the bond prices. After that, the VAR remains quite stable. The HS-100 VAR is more volatile since it uses a smaller sample. There is a jump in the VAR estimates in December 2001 due to a significant change in interest rates. The VAR estimate returns to lower levels once the December 6, 2001 observation is excluded from the historical distribution of the zero interest rates. The 3F-MC and 5F-MC VAR are greater and more volatile than the HS-252 VAR. In the 95% case, the VAR patterns are quite similar and they can be explained analogously to the 99% case.

[4] *Equation (3) can also be interpreted as a modified version of the process used in the Libor market models (for example, Brace, Gatarek & Musiela, 1997) since spot rather than forward rates are used*

[5] *Swap rates are converted to zero rates by following the bootstrapping technique described in Rebonato (1998)*

[6] *Many studies in the interest rate literature have found that the first three PCs explain more than 95% of the total variation in interest rate changes (see Wilson, 1994)*

B. Bond portfolio

	HS-100	HS-252	3F-MC	5F-MC
99% VAR				
Average VAR	€0.802m	€0.635m	€0.798m	€0.851m
No. of exceptions	2	4	1	1
Kupiec test	0.097	0.806	1.140	1.140
Kuiper test	0.004	0.017	0.020	0.024
95% VAR				
Average VAR	€0.493m	€0.403m	€0.565m	€0.603m
No. of exceptions	11	18	6	5
Kupiec test	0.161	2.399	4.208*	5.886*
Kuiper test	0.004	0.017	0.020	0.024

Back-testing the one-day 99% and 95% HS-100 VAR, HS-252 VAR, 3F-MC VAR and 5F-MC VAR. Results are reported for Kupiec and Kuiper tests. The back-testing period is 247 days, and the total value of the portfolio is €100 million. The critical values for Kupiec and Kuiper tests are 3.841 and 0.109, respectively. An asterisk denotes rejection of the method

Table B shows the results from the back-testing criteria for the various VAR methods applied to the bond portfolio case. The results are mixed depending on the criterion and on the confidence level. In the 99% case, both Kupiec and Kuiper tests accept the four VAR methods. On the other hand, in the 95% case the Kupiec test rejects the MC simulation methods, while the Kuiper test accepts all methods. In the 99% level, all methods fall in the 'green' BIS zone. In the 95% level, the variation of the number of exceptions is greater than the 99% level, just as it was the case in the stock analysis. The HS-252 method yields more exceptions than expected, while the MC methods deliver the smallest number of exceptions. However, the 5F-MC simulation yields the most conservative average VAR figures. This is true for both confidence levels. Finally, we can see that increasing the number of PCA factors does not improve the performance of the VAR model.

Conclusions and implications

We have investigated the accuracy of historical versus MC simulation for VAR calculation purposes (95% and 99% one-day VAR). The two approaches were applied to linear (stocks) and non-linear (bonds) portfolios. Greek data sets were used to examine the performance of the methods in turbulent markets. We employed various criteria (BIS, Kupiec and Kuiper) to back-test the methods and to decide on their accuracy.

The historical simulation was performed using different sample sizes. In the stock portfolio case, MC simulation was implemented using three different volatility/correlation estimators. In the bond portfolio case, the historical and MC simulation were performed on interest rate curves rather than directly on bond portfolio returns. The portfolio's P/L distribution was constructed using the simulated rates. Our contribution to the fixed-income VAR literature is twofold in terms of the chosen interest rate process, and the method – PCA – that is used to estimate the interest rate volatilities.

The results are mixed depending on the portfolio type, the criterion that is used and the confidence level. In particular, in the stock portfolio case, both Kupiec and Kuiper tests accept all methods. However, the historical simulation commits more capital than is necessary, and should therefore not be used.[7] On the other hand, in the bond portfolio analysis, no method is clearly superior since the results differ across tests and confidence levels. The latter finding extends Hendricks' (1996) result to non-linear portfolios. He had found that for linear portfolios, the reliability of the method depends on the choice of the confidence level. For both portfolios, the number of exceptions encountered (the BIS rule) depends on the sample size, as well as on the volatility/correlation estimator. Finally, we found that the choice of the number of PCA factors does not affect the performance of the VAR model. This is in contrast to the well-known fact that the number of the PCA factors is important for pricing and hedging applications (see Wilson, 1994, for a discussion).

Future research should compare the VAR and CVAR concepts by measuring market risk for various portfolios and then applying back-testing criteria. To the best of our knowledge, this comparison has been done only in the context of portfolio allocation where expected returns and VAR/CVAR efficient frontiers are constructed (see Gaivoronski & Pflug, 2002). ■

George Skiadopoulos is a lecturer in the department of banking and financial management at the University of Piraeus, and an associate research fellow in the Financial Options Research Centre at the University of Warwick. Greg Lambadiaris, Louiza Papadopoulou and Yiannis Zoulis are research associates in financial engineering. **They would like to thank Charalambos Kourtidis, Vasiliki Skintzi, Stavros Zenios, Nicholas Dunbar and two anonymous referees for many helpful discussions and comments.** **Email: gskiado@unipi.gr**

[7] *We also examined the VAR properties of two alternative stock portfolios that exhibit a greater and smaller volatility than the portfolio used in this article, respectively; the same portfolio structure is maintained. We found that as volatility increases, the average VAR figure increases as well, as expected. Furthermore, the back-testing criteria accept both methods, regardless of the level of volatility*

REFERENCES

Artzner P, F Delbaen, J Eber and D Heath, 1997
Thinking coherently
Risk November, pages 68–71

Beder T, 1995
VAR: seductive but dangerous
Financial Analysts Journal 51, pages 12–24

Brace A, D Gatarek and M Musiela, 1997
The market model of interest rate dynamics
Mathematical Finance 7, pages 127–155

Brenner R, R Harjes and K Kroner, 1996
Another look at models of the short-term interest rate
Journal of Financial and Quantitative Analysis 31, pages 85–107

Chan K, A Karolyi, F Longstaff and A Sanders, 1992
An empirical comparison of alternative models of the short-term interest rate
Journal of Finance 47, pages 1,209–1,247

Chapman D and N Pearson, 2001
Recent advances in estimating term-structure models
Financial Analysts Journal 57, pages 77–95

Crnkovic C and J Drachman, 1996
Quality control
Risk September, pages 138–143

Dothan U, 1978
On the term structure of interest rates
Journal of Financial Economics 6, pages 59–69

Dunbar N, 2002
Value-at-risk: down but not out
Risk November, page 121

Engle R and K Kroner, 1995
Multivariate simultaneous Garch
Econometric Theory 11, pages 122–150

Figlewski S, 1997
Forecasting volatility, financial markets, institutions and instruments
New York University Salomon Center, volume six

Gaivoronski A and G Pflug, 2002
Properties and computation of value-at-risk efficient portfolios based on historical data
Working paper, University of Vienna

Heath D, R Jarrow and A Morton, 1992
Bond pricing and the term structure of interest rates: a new methodology for contingent claims evaluation
Econometrica 60, pages 77–105

Hendricks D, 1996
Evaluation of value-at-risk models using historical data
Federal Reserve Bank of New York Economic Policy Review 2, pages 39–70

Jorion P, 2001
Value at risk: the new benchmark for controlling market risk
McGraw Hill, New York, second edition

JP Morgan, 1996
RiskMetrics technical document
Fourth edition

Kupiec P, 1995
Techniques for verifying the accuracy of risk measurement models
Journal of Derivatives 3, pages 73–84

Linsmeier T and N Pearson, 2000
Value-at-risk
Financial Analysts Journal 56, pages 47–67

Nowman K, 1997
Gaussian estimation of single-factor continuous time models of the term structure of interest rates
Journal of Finance 52, pages 1,695–1,706

Rebonato R, 1998
Interest-rate option models
John Wiley & Sons, second edition

Singh M, 1997
Value at risk using principal components analysis
Journal of Portfolio Management, fall, pages 101–112

Vlaar P, 2000
Value at risk models for Dutch bond portfolios
Journal of Banking and Finance 24, pages 1,131–1,154

Wilson T, 1994
Debunking the myths
Risk April, pages 67–73